Noise-enhanced transmission of information in a bistable system

M. Misono
Graduate School of Science and Technology, Kobe University, Nada, Kobe 657-0013, Japan

T. Kohmoto, Y. Fukuda, and M. Kunitomo
Department of Physics, Faculty of Science, Kobe University, Nada, Kobe 657-0011, Japan
(Received 1 June 1998)

Stochastic resonance (SR) in a bistable system with a binary input signal is studied by numerical simulation. The mutual information (MI) is used as a measure of information. We show that MI shows the resonance peculiar to SR and that the output MI of the bistable system can exceed the input one, that is, MI can have gain. The dependence of the resonance signal and the gain of MI on the noise cutoff frequency is studied and it is shown that the noise with large cutoff frequency is favorable for SR and for the gain of MI. To show the characteristics and the benefits of this type of SR and the gain of MI, the behavior of SR in the bistable system for image data is simulated. The simulation results show that this system can improve the visibility of images. [S1063-651X(98)00311-0]

PACS number(s): 05.40.+j, 89.70.+c

I. INTRODUCTION

Stochastic resonance (SR) is a phenomenon in which the transmission of a coherent signal is enhanced by adding random noise in some nonlinear systems. This phenomenon was first discussed in the study of the glacial period [1–3] and has been explored in various fields [4–6].

Most of previous studies on SR are performed in the systems with a sinusoidal input. Since a sinusoidal wave has no information, the results of these studies cannot be used in signal processing in biological or artificial systems. In addition, the conventional definition [4] of the signal-to-noise ratio (SNR) is valid only in systems with a sinusoidal input. Spectra of aperiodic signals that have finite information have finite width, thus the conventional definition of the SNR can hardly be applied.

In recent years, studies on SR with aperiodic input signals have been performed and definitions of measures of transinformation in the place of the conventional SNR were proposed [7–14]. Studies on neurophysiological systems using models with threshold nonlinearity with a binary input signal were performed [7,8]. In these studies, the mutual information (MI) was used as a measure. SR in a Schmitt trigger circuit was studied [9] and the dynamical entropy was used as a measure. SR in a bistable system with a binary input signal and white noise was studied [10] and the percentage of transmitted bits was used as a measure.

In the present paper the characteristics of a transmission line comprising a bistable system is studied from the information-theoretic standpoint. It is shown by numerical simulations employing MI as a measure with a binary input signal and colored noise that a transmission line with a bistable system can transmit more information than that without a bistable system. Since the simple model used in the present simulation involves the essentials of bistable systems, the present study can be generally applied to any studies on the behavior of transmission lines comprising bistable systems. Moreover, the numerical process in a computer by itself is proved to be a powerful tool in signal processing. MI is advantageous to measure the transinformation of binary-bit series and is utilized in the studies on neurophysiological systems mentioned above. In the present study MI is used to measure the transinformation of a binary signal in a bistable system and is proved to be advantageous also in this system.

In the systems in which the input signal is a sinusoidal wave and the conventional definition of the SNR is used, the output SNR does not exceed the input one [15]. This type of system is beneficial only when there are no means to detect the input signal. In contrast, this is not true for SR with a binary input signal. More information can be obtained from the output wave form than from the input one, that is, MI has gain, because the two states of binary bits are separated more distinctly due to the potential barrier in the bistable system. In the present study, this characteristic is proved using MI as a measure of transinformation and the applicability of SR in bistable systems to signal processing is demonstrated by transmitting image data.

The influence of noise cutoff frequency on SR in this system is also studied. In addition to the interest in fundamental characteristics, this subject is practically significant because instruments, such as a noise generator or an amplifier, have a limited bandwidth. Previous studies on the sinusoidal input type of SR systems [16–21] have concluded that noise with a large cutoff frequency is favorable for SR. In the present study we show that such noise is favorable also for SR with a binary input signal. The maximum value of the output MI and the maximum gain of MI become smaller as the cutoff frequency of the input noise becomes smaller. Furthermore, we show that MI comes to have no gain for small noise-cutoff frequency, thus the benefit of this type of SR to signal processing vanishes for this noise type.

II. SIMULATION

Mutual information was used as a measure of the transinformation in the present simulation. Here a bit in the input bit series x is high with probability p_x and is low with probability $p_{\bar{x}} = 1 - p_x$. Similarly a bit in the output bit series y is high with probability p_y and is low with probability $p_{\bar{y}}=1-p_y$. The conditional probability p_{yx} is that y is high under the condition that x is high and the other conditional probabilities are defined in a similar way. The mutual information $I(x,y)$ between the bit series x and y is defined as [7]

$$I(x,y)=H(y)-H(y|x), \tag{1}$$

$$H(y)=-p_y \log_2 p_y - p_{\bar{y}} \log_2 p_{\bar{y}}, \tag{2}$$

$$H(y|x)=p_x(-p_{yx}\log_2 p_{yx}-p_{\bar{y}x}\log_2 p_{\bar{y}x}) + p_{\bar{x}}(-p_{y\bar{x}}\log_2 p_{y\bar{x}}-p_{\bar{y}\bar{x}}\log_2 p_{\bar{y}\bar{x}}). \tag{3}$$

$H(y)$ is the entropy of y and $H(y|x)$ is the conditional entropy of y for given x. When $p_x=p_{\bar{x}}=p_y=p_{\bar{y}}=1/2$, for example, $I(x,y)$ takes the following values. If the output y is identical to the input x, that is, all of the information is transmitted, the conditional probabilities p_{yx} and $p_{\bar{y}\bar{x}}$ are unity, while $p_{\bar{y}x}$ and $p_{y\bar{x}}$ are 0; thus $I(x,y)$ is unity. On the other hand, if the output y has no correlation with the input x, that is, no information is transmitted, all of the conditional probabilities are 1/2; thus $I(x,y)$ is 0.

In the present simulation, the time evolution of the following equation was calculated:

$$\frac{dz_{out}(t)}{dt}=az_{out}(t)-b\{z_{out}(t)\}^3+z_{in}, \tag{4}$$

$$z_{in}=Ahs(t)+\xi(t), \tag{5}$$

$$h=\sqrt{\frac{4a^3}{27b}}, \tag{6}$$

where z_{out} and z_{in} are the output and the input wave forms of the bistable system, A is the signal amplitude, $s(t)$ is the original bit series, and $\xi(t)$ is the input noise. The terms $az_{out}-bz_{out}^3$ are the contributions from the bistable potential.

For $s(t)$ we used the pseudorandom bit series (PRBS) (maximum-length linear shift register sequence) with a period of $2^{15}-1$ bits (PRBS $2^{15}-1$). Each bit of $s(t)$ takes the value $+1$ (high) or -1 (low). When $A=1$ with the factor $h=\sqrt{4a^3/27b}$, one of the two minima of the distorted bistable potential $-(a/2)z_{out}^2+(b/4)z_{out}^4+hz_{out}$ coincides with its maximum. When h takes this value, interwell motion is possible without noise when $A>1$ and is impossible without noise when $A\leqslant 1$.

The noise $\xi(t)$ is the Ornstein-Uhlenbeck (OU) noise [18]

$$\frac{d\xi(t)}{dt}=-\frac{1}{\tau}\xi(t)+\frac{\sqrt{D}h}{\tau}\xi_w(t), \tag{7}$$

where τ is the correlation time of the noise, D is the noise intensity, and $\xi_w(t)$ is the white noise. The noise cutoff frequency f_c is

$$f_c=\frac{1}{2\pi\tau}. \tag{8}$$

In the following, f_c is specified in units of the bit rate R. The OU noise in the present simulation has a Gaussian amplitude distribution with zero mean. The standard deviation of the white noise in Eq. (7) is unity.

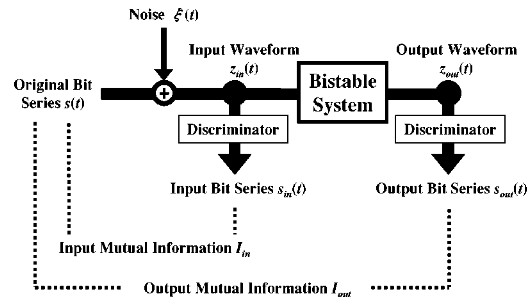

FIG. 1. Schematic diagram of the simulation.

The procedure of the simulation is shown in Fig. 1. From the calculation of the time evolution of Eqs. (4) and (5) for 32×10^3 bits of a PRBS, the input wave form $z_{in}(t)$ and the output wave form $z_{out}(t)$ are obtained. In this calculation, 100 steps of calculations are implemented in the time interval corresponding to a single bit. Then these wave forms are discriminated to obtain the input bit series $s_{in}(t)$ and the output bit series $s_{out}(t)$. The input and output wave forms are discriminated at the 50th step and at the last step of each bit, respectively. The bit series $s_{in}(t)$ and $s_{out}(t)$ are compared with the original one $s(t)$; thus the input mutual information $I_{in}=I(s(t),s_{in}(t))$ and the output MI $I_{out}=I(s(t),s_{out}(t))$ are obtained.

III. RESULTS

Figure 2 shows the relation between the output mutual information I_{out} and the noise intensity D for various signal amplitudes A. The values of a and b in Eq. (4) are 18 and 30 and the ratio of f_c and the bit rate R of $s(t)$, f_c/R, is 10. The curves are for $A=0.2-2.0$ in steps of 0.2 in increasing order. When $D=0$, I_{out} is 0 for $A\leqslant 1$ and is unity for $A>1$. This behavior of I_{out} is expected from the definition of A described in Sec. II.

For $A\leqslant 1$, the resonancelike enhancement of I_{out} peculiar to SR appears with increasing D. For small values of D,

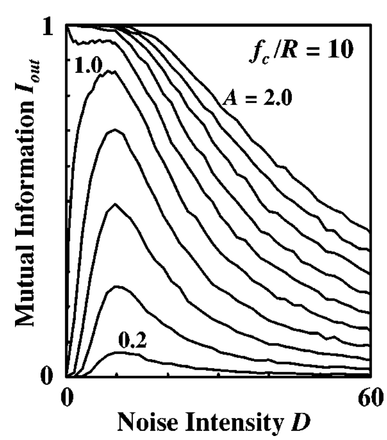

FIG. 2. Dependence of the resonance curve on the signal amplitude A. The abscissa is the noise intensity D and the ordinate is the output mutual information I_{out}. The curves are for $A=0.2-2.0$ in steps of 0.2 in increasing order. The ratio $f_c/R=10$ and the parameters $a=18$ and $b=30$ in Eq. (4).

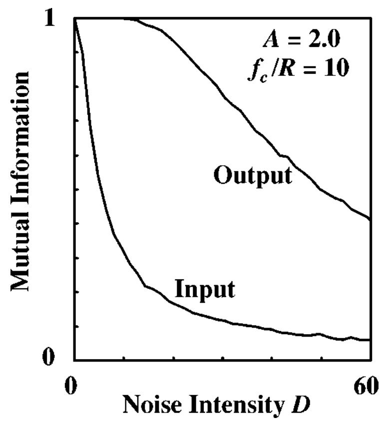

FIG. 3. Relation between the output mutual information and the noise intensity D and that between the input one and D. The ratio $f_c/R=10$ and the signal amplitude A is 2.0.

since the input wave form $z_{in}(t)$ causes almost only the intrawell motion, the output bit series $s_{out}(t)$ has long consecutive series of high bits or of low bits. In particular, when $D=0$, $s_{out}(t)$ consists of only high bits or low bits. When the value of D is adequate, $s_{out}(t)$ reproduces the original bit series $s(t)$ relatively well and I_{out} achieves its maximum. Larger values of D reduce I_{out}.

For $A>1$, the resonancelike enhancement does not appear. When $D=0$, I_{out} is unity, that is, all the information is transmitted through the bistable system without noise. While the value of D remains small, I_{out} still holds the value near unity because the noise causes almost only the intrawell motion. Larger values of D reduce I_{out} like for $A\leqslant 1$.

Figure 3 shows the relation between MI and the noise intensity D when $A=2.0$ and $f_c/R=10$, where I_{in} and I_{out} are shown. Both I_{in} and I_{out} decrease monotonically as the value of D increases. For small values of D, I_{out} holds the value near unity, while I_{in} decreases rapidly. For all values of D shown in this figure, I_{out} has a larger value than I_{in}. This shows that the bistable system reduces the degradation of the signal when $A>1$.

In the following, the simulations are performed with $A=0.8$. Figure 4 shows the relation between the mutual information and the noise intensity D when $f_c/R=10$, where I_{in} and I_{out} are shown. Increasing the value of D, I_{in} decreases monotonically. When $D>2.2$, I_{out} is larger than I_{in}. In this region, more information is obtained by transmitting the bit series buried in the noise through the bistable system.

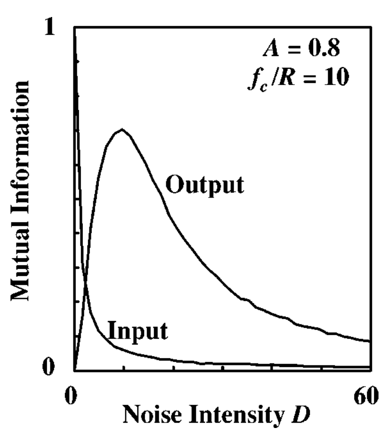

FIG. 4. Relation between the output mutual information and the noise intensity D and that between the input one and D. The ratio $f_c/R=10$ and the signal amplitude A is 0.8.

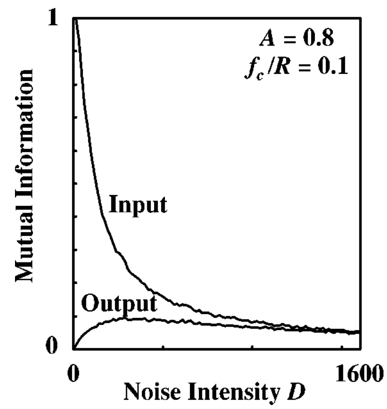

FIG. 5. Relation between the output mutual information and the noise intensity D and that between the input one and D. The ratio $f_c/R=0.1$ and the signal amplitude A is 0.8. Note that the maximum value of D is about 27 times as large as that in Fig. 4.

Figure 5 shows the relations between the MI and the noise intensity D when $f_c/R=0.1$. In this figure the range of D is from 0 to 1600, about 27 times larger than that in Fig. 4. Since I_{in} does not suffer much degradation from the noise, it decreases slowly as the value of D is increased. I_{out} shows the resonancelike behavior. However, the maximum value of I_{out} is small, the value of D at the maximum I_{out} is large, and the width of the resonance curve is large. For small values of f_c/R, since the change of the noise in the duration of a single bit is small, the noise causes the interwell motion less frequently even when the value of D is optimum. Therefore, I_{out} shows less remarkable resonance. In the range shown in Fig. 5, I_{out} does not exceed I_{in}. The gain of the transinformation cannot be expected when the input noise is the OU noise with a small value of f_c. When the noise amplitude is much larger than the signal amplitude and the potential barrier, the influence of the potential barrier becomes smaller.

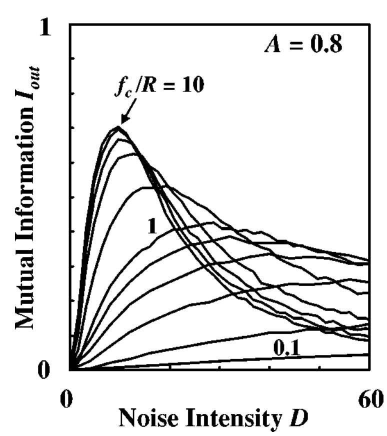

FIG. 6. Dependence of the resonance curve on the noise cutoff frequency f_c. The relation between the output mutual information I_{out} and the noise intensity D is shown. The ratio f_c/R is 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 2.0, 4.0, 6.0, 8.0, and 10.0 in increasing order. The signal amplitude A is 0.8.

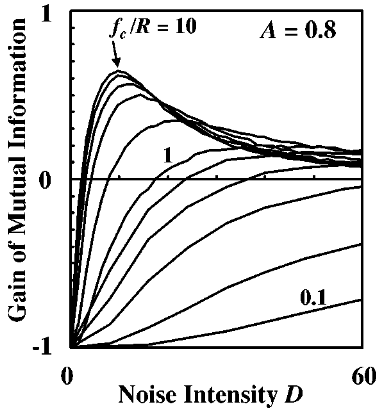

FIG. 7. Dependence of the gain of mutual information (GMI), $I_{out}-I_{in}=H(s_{in}(t)|s(t))-H(s_{out}(t)|s(t))$, on the noise cutoff frequency f_c. The ratio f_c/R is 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 2.0, 4.0, 6.0, 8.0, and 10.0 in increasing order. The signal amplitude A is 0.8. The positive value of GMI means that more information is obtained from the output of the bistable system than from the input.

The dependence of the resonance curve on the noise cutoff frequency f_c is shown in Fig. 6. The relation between the output mutual information I_{out} and the noise intensity D is shown for $f_c/R=0.1$, 0.2, 0.4, 0.6, 0.8, 1.0, 2.0, 4.0, 6.0, 8.0, and 10.0 in increasing order. For a larger noise cutoff frequency f_c, as in the case of SR with a sinusoidal input signal, the maximum value of I_{out} is larger, the value of D at the maximum I_{out} is smaller, and the width of the resonance curve is smaller.

Figure 7 shows the gain of mutual information (GMI) $I_{out}-I_{in}=H(s_{in}(t)|s(t))-H(s_{out}(t)|s(t))$ for the same values of f_c/R as in Fig. 6 in increasing order. The positive value of the GMI means that more information is obtained from the output of the bistable system than from the input. For a larger value of f_c, the maximum value of the GMI is larger, the value of D at the maximum GMI is smaller, and the GMI becomes positive at a smaller value of D. When the ratio f_c/R is small, MI hardly has gain, and when $f_c/R=0.1$, no gain is obtained, as is described in the above.

IV. DISCUSSION

The feature of this type of SR is that the output mutual information I_{out} can exceed the input MI I_{in}, that is, more information is obtained by transmitting bit series through the bistable system. Under the conditions in Fig. 4, when $D=10$, I_{in} is 0.06, thus only a small amount of information is obtained if the wave form is received before transmitting the bistable system. If the wave form is received after transmitting the bistable system, on the other hand, the mutual information I_{out} is 0.70, thus a larger amount of information can be obtained.

When $D=2$ in Fig. 4, for example, transmitting the wave form through the bistable system in itself results in the degradation of MI. However, if the noise is added to the input wave form to reach $D=10$ and next the wave form is transmitted through the bistable system, I_{out} exceeds I_{in}. Adding more noise and transmitting through the bistable system can improve the transmission characteristic; this paradoxical but profitable phenomenon appears in the range $0.7<D<10$. The larger improvement is possible for bit series buried in the noise with larger cutoff frequency by this procedure. When the ratio f_c/R is small, since I_{out} hardly exceed I_{in} as shown in Fig. 7, the gain of transinformation cannot be expected.

The gain of MI by SR becomes quite obvious by simulating the behavior of the input and output data when a binary-bit series of image data is inputted. This simulation is realized by substituting the image data for PRBS $2^{15}-1$ in the simulation described in Sec. II. In this simulation we used a 256-color windows-bit-map file of 48×10^3 bytes as input image data. We converted the bit-map file to a bit series of $8\times48\times1024$ bits. The input wave form of the bistable system is the sum of this bit series and the OU noise. The wave form is transmitted through the bistable system and the output wave form is obtained. For the input and output wave forms, each bit is discriminated as described in Sec. II. Image files are reproduced from the discriminated bit series. In the present study, the header bytes of the image files are removed before the simulation and are attached to the discriminated bit series when the image files are reproduced.

The reproduced bit-map images for $D=0$, 1.6, 10, and 60 are shown in Fig. 8. The images obtained from the input wave forms are shown on the left and those obtained from the output wave forms are shown on the right, where $f_c/R=10$. The input image for $D=0$ is identical to the original one. The input image becomes less visible monotonically as the value of D is increased. The visibility of the output images shows resonancelike behavior corresponding to the I_{out} curve in Fig. 4. When $D=0$, since the input wave form causes in the bistable system only the intrawell motion, the discriminated output bit series consists of only high bits or low bits. Thus all pixels in the image are white if all bits are high and black if all bits are low as in the present simulation. Whether the pixels are all black or all white depends on the initial conditions. When $D=1.6$, though I_{out} is nearly equal to I_{in}, the appearances of the images are different. Since the output wave form consists of long series of high bits or low bits, white pixels and black pixels are dominant, while the input image is randomized by the noise. When $D=10$, the optimum noise intensity case, the output image is distinct enough to grasp the content of the image. Adding more noise, when $D=60$, the output image becomes less visible and is similar in appearance to the input one.

The improvement of the visibility of images by transmitting through the bistable system can be realized by comparing the input and output images when $D>2.2$. The improvement of the visibility of images by adding more noise and transmitting through the bistable system can be realized when $0.7<D<10$ as follows. Consider, for example, a wave form with some noise that generates the input image of $D=1.6$. Adding more noise to the wave form until the value of D becomes 10 and transmitting it through the bistable system, one obtains the more distinct image.

The images for $D=0$, 288, and 1600 when the input noise is the OU noise with $f_c/R=0.1$ are shown in Fig. 9. The input image becomes less visible as the value of D is increased, monotonically and more slowly than the images for $f_c/R=10$. The output images, even for large value of D, appear like the image for $D=1.6$ in the case of $f_c/R=10$ because of the small rate of interwell motion. It cannot be

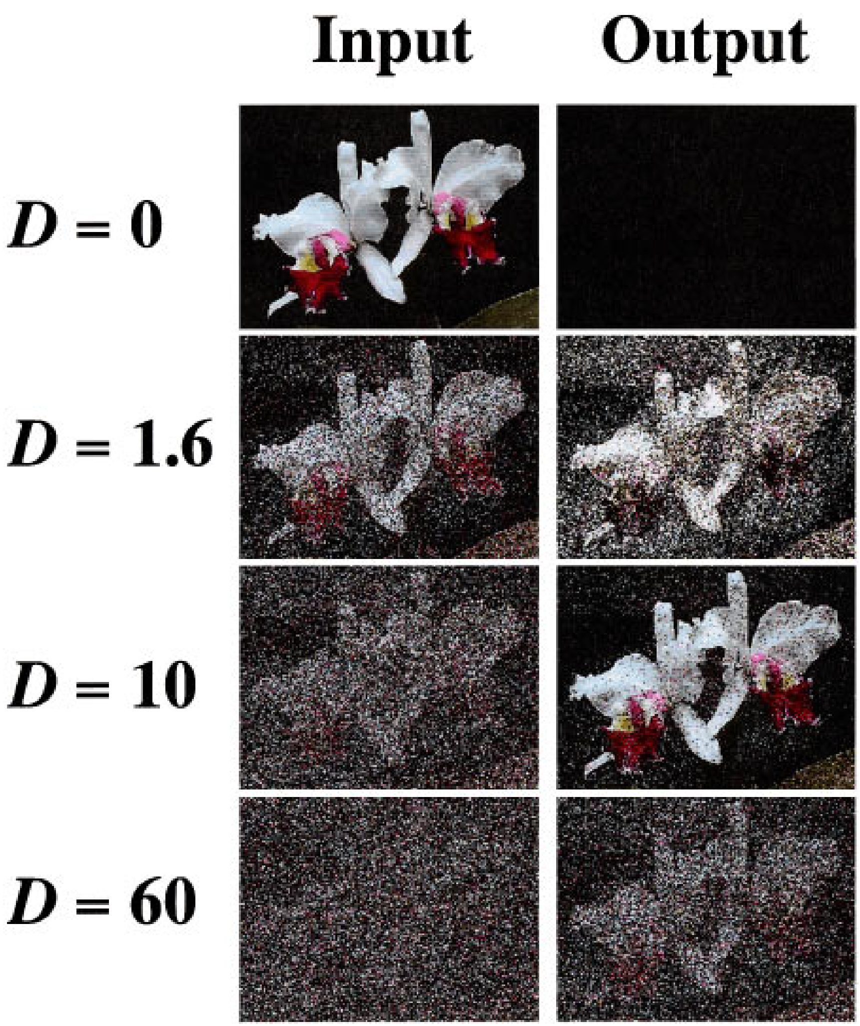

FIG. 8. (Color) Reproduced bit-map images for $D=0$, 1.6, 10, and 60 when $f_c/R=10$. The images obtained from the input wave forms are shown on the left and those obtained from the output wave forms are shown on the right. The input image for $D=0$ is identical to the original one.

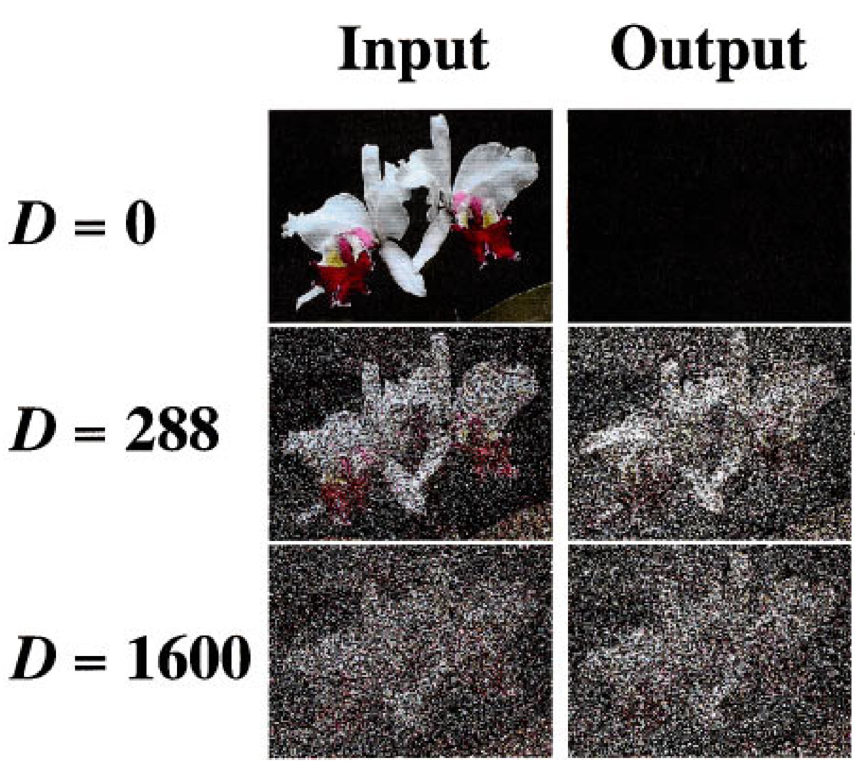

FIG. 9. (Color) Reproduced bit-map images for $D=0$, 288, and 1600 when $f_c/R=0.1$. The images obtained from the input wave forms are shown on the left and those obtained from the output wave forms are shown on the right. The input image for $D=0$ is identical to the original one.

said that the output images are more distinct than the input ones. The OU noise with small cutoff frequency relative to the bit rate cannot improve the visibility of the images.

V. CONCLUSION

Numerical simulations of SR when binary-bit series are inputted to a bistable system were performed. I_{out} shows resonance peculiar to SR when $A \leqslant 1$; when $A > 1$, instead of showing the resonance, the bistable system reduces the degradation of the signal. The output MI can exceed the input MI. The noise with large cutoff frequency is favorable for SR and the gain of MI. This system can improve the visibility of images and this means that this system is a powerful tool for signal processing.

ACKNOWLEDGMENTS

We would like to thank the Green House in Faculty of Agriculture, Utsunomiya University, for permission to use the pictures. This work was supported by a Grant-in-Aid from the Ministry of Education, Science and Culture of Japan.

[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981).

[2] R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, Tellus **34**, 10 (1982).

[3] C. Nicolis, Tellus **34**, 1 (1982).

[4] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).

[5] Proceedings of International Workshop on Fluctuations in Physics and Biology: Stochastic Resonance, Signal Processing, and Related Phenomena, edited by R. Mannella, P. V. E. McClintock, and A. Bulsara [Nuovo Cimento D **17**, 661 (1995)].

[6] Proceedings of the NATO Advanced Research Workshop on Stochastic Resonance in Physics and Biology, edited by F. Moss, A. Bulsara, and M. F. Schlesinger [J. Stat. Phys. **70**, 1 (1993)].

[7] A. R. Bulsara and A. Zador, Phys. Rev. E **54**, R2185 (1996).

[8] F. Chapeau-Blondeau, Phys. Rev. E **55**, 2016 (1997).

[9] A. Neiman, B. Shulgin, V. Anishchenko, W. Ebeling, L. Schimansky-Geier, and J. Freund, Phys. Rev. Lett. **76**, 4299 (1996).

[10] G. Hu, D. Gong, X. Wen, C. Yang, G. Qing, and R. Li, Phys. Rev. A **46**, 3250 (1992).

[11] C. Heneghan, C. C. Chow, J. J. Collins, T. T. Imhoff, S. B. Lowen, and M. C. Teich, Phys. Rev. E **54**, R2228 (1996).

[12] J. J. Collins, C. C. Chow, and T. T. Imhoff, Phys. Rev. E **52**, R3321 (1995).

[13] J. J. Collins, C. C. Chow, and T. T. Imhoff, Nature (London) **376**, 236 (1995).

[14] A. Neiman and L. Schimansky-Geier, Phys. Rev. Lett. **72**, 2988 (1994).

[15] M. I. Dykman, D. G. Luchinsky, R. Mannella, P. V. E. McClintock, N. D. Stein, and N. G. Stocks, Nuovo Cimento D **17**, 661 (1995).

[16] M. Misono, T. Kohmoto, Y. Fukuda, and M. Kunitomo, Opt. Commun. **152**, 255 (1998).

[17] R. N. Mantegna and B. Spagnolo, Nuovo Cimento D **17**, 873 (1995).

[18] P. Hänggi, P. Jung, C. Zerbe, and F. Moss, J. Stat. Phys. **70**, 25 (1993).

[19] T. Zhou and F. Moss, Phys. Rev. A **41**, 4255 (1990).

[20] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta, and S. Santucci, Phys. Rev. Lett. **62**, 349 (1989).

[21] L. Gammaitoni, E. Menichella-Saetta, S. Santucci, F. Marchesoni, and C. Presilla, Phys. Rev. A **40**, 2114 (1989).